INDEFINITE INTEGRATION

Basic Integration	
$\int 0.dx = c$	$\int 1.dx = x + c$
$\int k.dx = kx + c$	$\int e^x dx = e^x + c$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$	$\int \frac{1}{x} dx = \log_e x + c$
$\int \sin x\, dx = -\cos x + c$	$\int \cos x\, dx = \sin x + c$
$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$	
$\int a^x dx = \frac{a^x}{\log_e a} + c = a^x \log_a e + c$	
$\int \frac{c}{ax+b} dx = \frac{c}{a} \log \lvert ax+b \rvert + c$	
$\int \log x\, dx = x \log x - x + c$	
$\int \log_a x\, dx = x \log_a x - \frac{x}{\log a} + c$	

Basic Theorems

$\int k\, f(x)dx = k\int f(x)dx$	$\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$

$$\int\left[f(x) \pm g(x)\right]dx = \int f(x)dx \pm \int g(x)dx$$

Methods of Integration

Integration by Substitution

If the integral is of the form $\int f(g(x)).g'(x)dx$, then we put $g(x) = t$ so that $g'(x)dx=dt$. Now integral is $\int f(t)dt$.

Integrand is the Product of Function and its Derivative

The integral is of the form $I= \int f'(x)\, f(x)dx$ we put $f(x) = t$ and convert it into a standard integral

Integrand is a Function of the Form f(ax+b)

We put ax+b =t and convert it into a standard integral

Integration of the form $\int\{f(x)\}^n f'(x)dx$	$\int\{f(x)\}^n f'(x)dx = \frac{\int\{f(x)\}^{n+1}}{n+1}$

Integration by Parts

$$\int(u.v)dx = u\left(\int v\, dx\right) - \int\left(\frac{du}{dx}\right)\left(\int v\, dx\right)dx$$

- The preference of selecting the u function will be according to the order ILATE (Inverse circular, logarithmic, Algebraic, Trigonometric, Exponential)

Two Classic integrals

- $\int e^x (f(x) + f'(x))\, dx = e^x.f(x) + c$
- $\int (f(x) + x.f'(x))\, dx = x.f(x) + c$

Integration of Rational Functions

In R(x)/g(x), factorize g(x) and then write partial fractions

1. Non-repeated linear factor in the denominator

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

2. Every repeated linear factor in the denominator.

$$\frac{1}{(x-a)^3(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3} + \frac{D}{(x-b)}$$

3. Non-repeated quadratic factor in the denominator

$$\frac{1}{(ax^2+bx+c)(x-d)} = \frac{Ax+B}{ax^2+bx+c} + \frac{C}{x-d}$$

4. Repeated quadratic factor in the denominator.

$$\frac{1}{(ax^2+bx+c)^2(x-d)} = \frac{Ax+B}{(ax^2+bx+c)^2} + \frac{Cx+D}{ax^2+bx+c} + \frac{E}{x-d}$$

Integration of irrational algebraic functions	
$\int \frac{1}{(ax+b)\sqrt{cx+d}}dx$	$\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$
$cx + d = z^2$	$px + q = z^2$
$\int \frac{dx}{(px+q)\sqrt{ax^2+bx+c}}$	$\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$
$px + q = 1/z$	$X = 1/z$
$\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(\beta-x)}}$	$\frac{dx}{(x-\alpha)\sqrt{(x-\beta)}}$
$x = a\cos^2 q + b\sin^2 q$	$x = a\sec^2 q - b\tan^2 q$

SPECIAL TRIGNOMETRIC FUNCTIONS

$\int \frac{dx}{a+b\sin^2 x}$	$\int \frac{dx}{a+b\cos^2 x}$	$\int \frac{dx}{(a\sin x+b\cos x)^2}$

$$\int \frac{dx}{a\cos^2 x + b\sin x\cos x + c\sin^2 x}$$

- Divide the numerator and denominator by **$\cos^2 x$** in all such types of integrals and then put **$\tan x = t$**

$\int \frac{dx}{a+b\cos x}$	$\int \frac{dx}{a+b\sin x}$	$\int \frac{dx}{a\cos x+b\sin x}$

- In such types of integrals we use the following substitutions

$$\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)} = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)} = \frac{1-t^2}{1+t^2};\ dx = \frac{2dt}{1+t^2}$$

$\int \frac{p\sin x+q\cos x}{a\sin x+b\cos x}dx$	$\int \frac{p\sin x}{a\sin x+b\cos x}dx$

- Numerator = A (denominator) + B (derivative of denominator)
- Then integral = Ax + B log (denominator) + C

Standard Substitutions

• $\sqrt{a^2-x^2}$ or $\frac{1}{\sqrt{a^2-x^2}}$	$x = a\sin\theta = a\cos\theta$
• $\sqrt{x^2+a^2}$ or $\frac{1}{\sqrt{x^2+a^2}}$	$x = a\tan\theta = a\cot\theta = a\sinh\theta$
• $\sqrt{x^2-a^2}$ or $\frac{1}{\sqrt{x^2-a^2}}$	$x = a\sec\theta = a\,\text{cosec}\,\theta$
• $\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$ or $\sqrt{x(a+x)}$ or $\frac{1}{\sqrt{x(a+x)}}$	$x = a\tan^2\theta$
• $\sqrt{\frac{x}{a-x}}$ or $\sqrt{\frac{a-x}{x}}$ or $\sqrt{x(a-x)}$ or $\frac{1}{\sqrt{x(a-x)}}$	$x = a\sin^2\theta$ $= a\cos^2\theta$
• $\sqrt{\frac{x}{x-a}}$ or $\sqrt{\frac{x-a}{x}}$ or $\sqrt{x(x-a)}$ or $\frac{1}{\sqrt{x(x-a)}}$	$x=a\sec^2\theta$ $=a\,\text{cosec}^2\,\theta$

Standard Integrals

- $\int \tan x\,dx = \log|\sec x| + c = -\log|\cos x| + c$
- $\int \cot x\,dx = \log|\sin x| + c = -\log|\text{cosec}\,x| + c$
- $\int \sec x\,dx = -\log|\sec x + \tan x| + c$
- $\int \text{cosec}\,x\,dx = -\log|\text{cosec}\,x + \cot x| + c$
- $\int \sec x \tan x\,dx = \sec x + c$
- $\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + c$
- $\int \sec^2 x\,dx = \tan x + c$
- $\int \text{cosec}^2 x\,dx = -\cot x + c$

Standard Substitutions

$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|+c$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+c$
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c$
$\int \frac{1}{\sqrt{x^2+a^2}}dx = \sinh^{-1}\left(\frac{x}{a}\right)$
$\int \frac{1}{\sqrt{x^2-a^2}}dx = \cosh^{-1}\left(\frac{x}{a}\right)+c$
$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$
$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left\|x+\sqrt{x^2+a^2}\right\|+c$
$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left\|x+\sqrt{x^2-a^2}\right\|+c$
$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$